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LETTER TO THE EDITORS

AN ANALYSIS OF LAMINAR COMBINED FORCED AND FREE CONVECTION HEAT TRANSFER IN A HORIZONTAL TUBE

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FARIS and Viskanta [1] recently reported an interesting perturbation solution for fully developed laminar flow in uniformly heated tubes, which includes the effects of secondary flow produced by buoyant forces for the horizontal orientation. As indicated by the authors, this is a problem of considerable practical interest since the average Nusselt numbers for combined forced and free convection can be substantially higher than the values indicated by the traditional constant property solutions. It is acknowledged that the solution is only approximate due to a number of restrictive assumptions; however, certain of the results and conclusions require comment since the information given in the paper will undoubtedly be applied to practical situations.

Consider first the established physical aspects cited in the Introduction. It does not appear that the magnitude of the free convection effect bears an inverse relation to the Prandtl number. Shannon and Depew [2] have recently shown that fully developed Nusselt numbers for ethylene glycol are substantially higher than those for water. Regarding the statement that there are no appreciable free convection effects in the thermal entrance region, it has been conclusively demonstrated by Petukhov and Polyakov [3] that fully developed flow can be established well upstream of the point where the flow is fully developed according to the constant property solution. The long starting lengths inferred from the authors' reference [7] have been misinterpreted. For Grashof numbers greater than 1000, the flow was generally developed at $Z/D < 100$; the subsequent rise in Nusselt number was a result of the increase in Grashof number with axial position due to the bulk temperature increase.

The final results given in Fig. 11 are typical of perturbation solutions in general in that the average Nusselt numbers blow up at moderate values of the perturbation parameter (moderate Grashof number). Although the finite-difference solution is quite involved, it does give much more accurate predictions of the heat-transfer coefficient for the large Grashof numbers of practical interest [4]. Another disturbing feature of Fig. 11 is that the Nusselt number is dependent on Reynolds number. This is contrary to what one would expect for a fully developed flow situation. For example, no evidence for a Reynolds number effect can be found in the authors' references [2, 7, 10] or in the present citations [1–4].

Finally, it is suggested that there is no appreciable circumferential wall temperature variation for most practical situations. This appears to be true for air (authors' references [2, 8]); however, the rather extensive data for water indicate that large circumferential gradients are obtained in practice. In an early study, Ede [5] noted a "substantial" temperature difference between top and bottom of his relatively thick-walled aluminum–brass test pipes. The data of Petukhov *et al.* [3, 6], which are given in detail, exhibit up to a 63°F temperature difference between top and bottom of the stainless test tube. This suggest that the assumption of constant tube wall temperature around the periphery used in the analysis is not generally valid. The boundary conditions of constant wall temperature and uniform heat flux give a solution which probably represents the upper limit of the heat-transfer coefficient, in the range where the perturbation solution is accurate. Another implication is that experimental tubes must be heavily instrumented with thermocouples if accurate average Nusselt numbers are to be derived.

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